Analysis, detailing and construction of a free-standing staircase

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The paper comprehensively gives information regarding analysis, detailing and construction of a free-standing staircase in a public building.

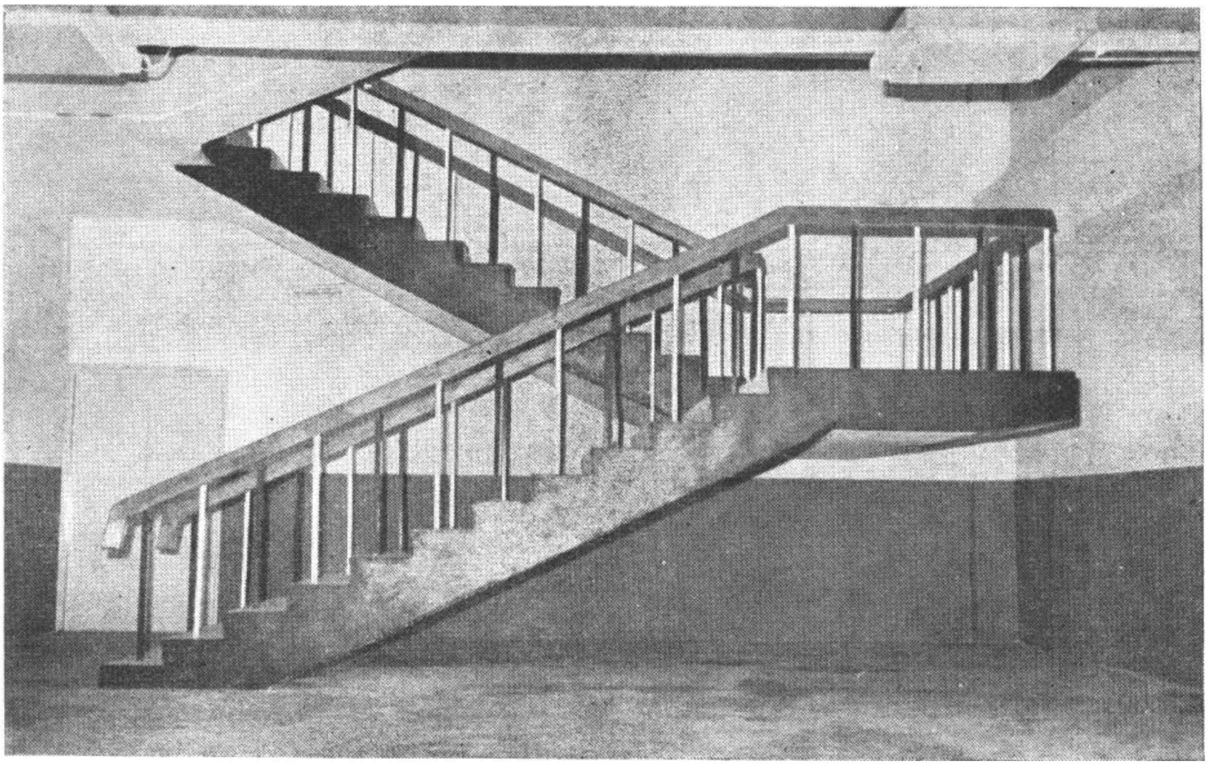

Fig 1 A view of the completed staircase

Published literature on the subject, so far has dealt mainly with the analysis of the free-standing staircase. No details of practical importance have been written about, particularly the design and detailing of the top landing in a multiflight staircase which is subjected to severe torsional stresses, apart from the normal flexural stresses. Similarly, the detailing of the foundation of the bottom flight creates a headache for the designer in practice. The author, himself been in such a predicament presents to other workers in the field his views on the subject.

Analysis

A free-standing staircase is a complicated structure. Though complete analysis is possible using numerical techniques likes finite difference or finite element methods, such methods of analysis are beyond the scope of the majority of design offices, since the stairs, constitute but a small item in the overall components of the building, both physically and financially. Yet, a well designed staircase as an architectural feature has a unique charm about it compared to any other part of the building *Figs* 1 and 2 and 3.

Detailing

Principles of detailing of reinforcemnt : The deflection studies for different loading conditions, *viz.* when only the lower flight is loaded, when only the landing is loaded and when all the portions of the staircase are loaded, as given by Chandrasekhara and Srinivasan, *Fig* 4[2]. This gives a very good practical insight into the desirable detailing of the reinforcement. Most of the experimental studies has led to the desirability of strengthening the midlanding as a beam element across the junction of the flights and the landing, as such studies have revealed that the stresses along the junction are non-linear and occur in high concentrations at the comers.

Based on the test results on a half-size concrete model, Sreenivasa Iyer and Manoharan have suggested that nominal torsional reinforcements should be given in the form of closed hoops for half the width of the midlanding, and that the cantilever reinforcements in the landing can be advantageously carried into the flights[2].

Venkateswarlu et al. have drawn the following conclusions from their investigation on torsional behaviour of reinforced concrete beams, which have a bearing on the detailing of reinforcement in the subject structure[5]. Their tests have indicated that

(i) the presence or absence of any amount of stirrup reinforcement is of no consequence to the torsional strength or ductility, if the top steel is not provided

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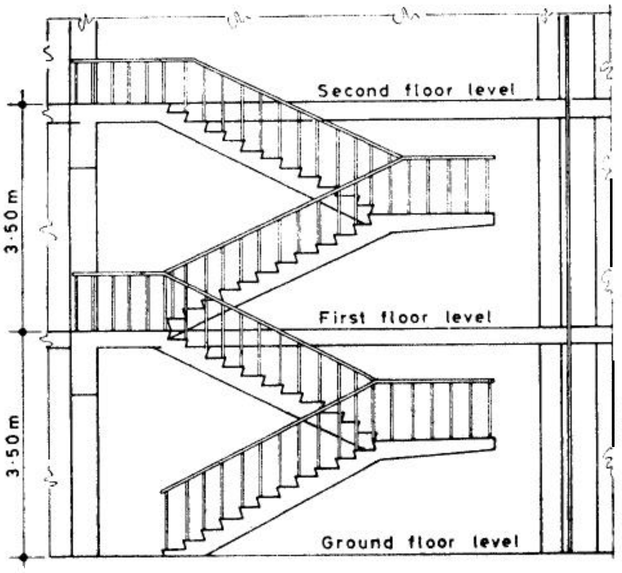
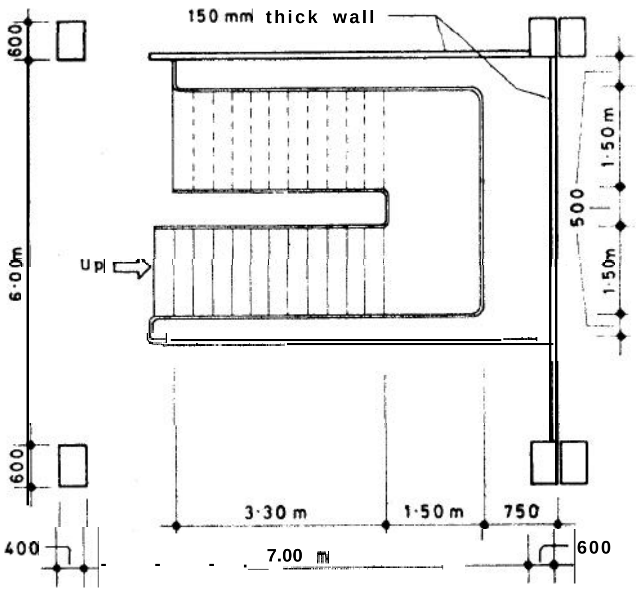


Elevation of staircase

Fig 2 Elevation of Staircase



Plan of stair case

Fig 3 Plan of staircase

(ii) the presence of the **bottom** longitudinal reinforcement will not add to the torsional strength of a plain concrete beam in the absence of the top longitudinal steel

(iii) the provision of the top longitudinal steel alone without bottom longitudinal steel is not useful, as the bottom longitudinal steel is required to take flexure in addition to torsion

(iv) the torsional reinforcement must consist of closely spaced stirrups and longitudinal bars

(v) the ductility of the beams without top **steel** is so small, that adequate warning is not available before failure.

Hence, from the above it can be seen that a torsion member has to have top and bottom reinforcement and closed stirrups, which enhances the torsional capacity of the members. Accordingly the reinforcement which was detailed in the staircase, is given in detail in the *Appendix* and shown in *Figs 5* to 8.

Construction

The construction of this staircase did not call for any special expertise, save for the necessity of rigid quality control during concreting and ensuring that the reinforcement is kept in its proper place. However, the following points are stressed for the guidance of site engineerrs

(i) entire staircase above construction joint shown at the beginning of the flight. at ground level, upto, and including the landing and landing beams on three sides, shall be concreted in one operation

(ii) all concrete is of M20 grade except the concrete in foundation, which is of M15 grade

(iii) props and shuttering shall not be removed before 28 days

(iv) while concreting the flights above first floor level, the flights in ground floor, shall be properly supported. if they are to be used for supporting the top flights

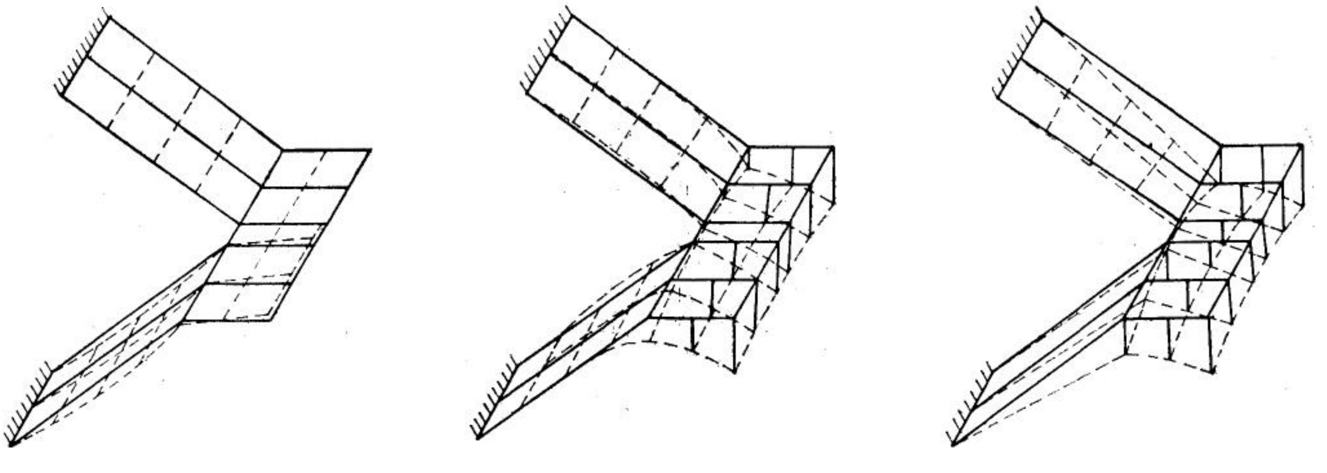


Fig 4 Deflections ‘in the, staircase when (a) only the lower flight is loaded *(b)* only the landing is loaded *(c)* all the parts of the staircase are loaded

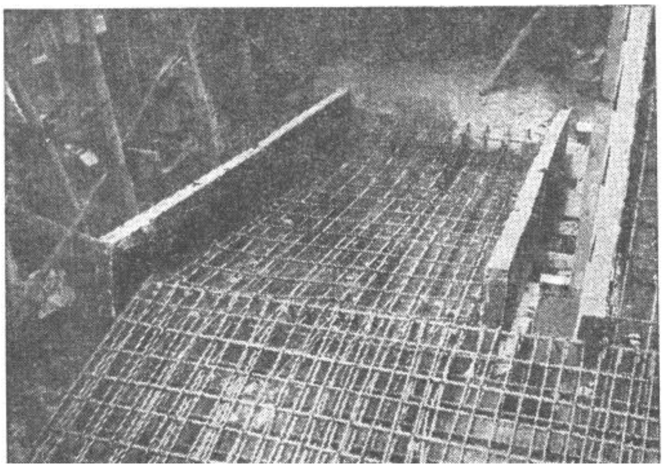


Fig 5 Reinforcement in first flight of stairs

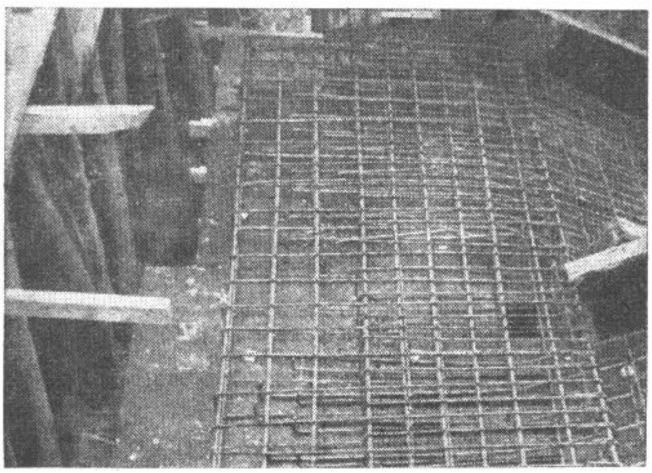


Fig 6 Remforcement in mid-landing

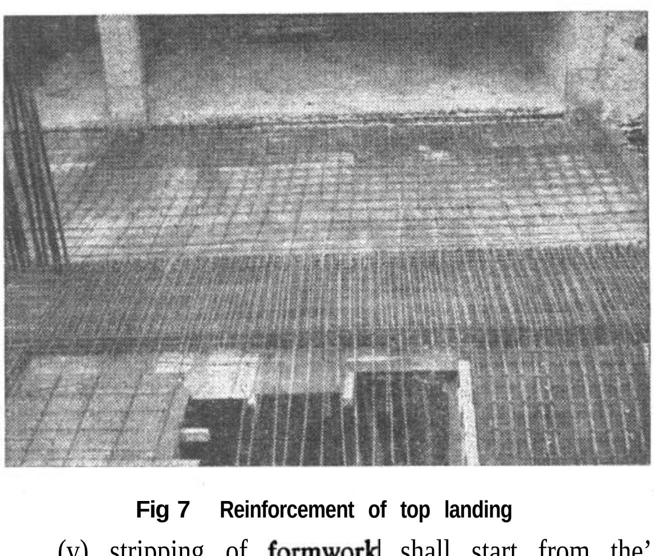


Fig 7 Reinforcement of top landing

(v) stripping of formwork shall start from the free edge of mid-landing and proceed towards both supports

(vi) at all stages of construction, the staircase shall be treated as a cantilever as a whole

(vii) necessary pockets for fixing balusters, etc. shall be preformed during concreting and no pockets should be left for after the concreting..

From the above, it can be seen that the forms and props get tied up for, about a month but that is more

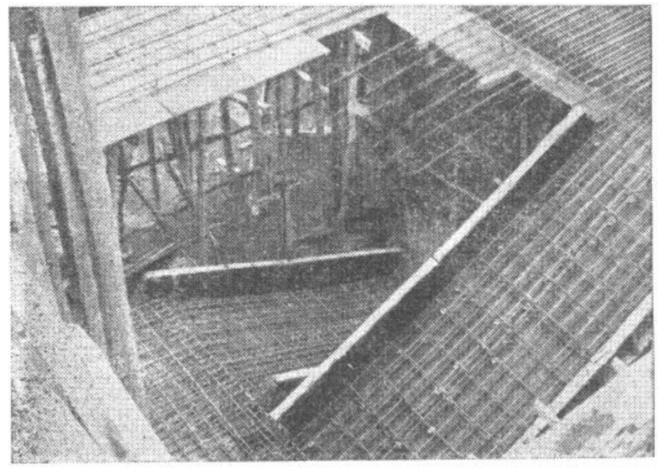


Fig 8 Reinforcement in entire staircase

than compensated by the final visual attraction of such a staircase.

Construction costs

The quoted cost for the construction of this staircase, at 1980 prices prevailing at Visakhapatnam, came to

1: 1½ : 3 reinforced concrete exclusive of formwork	Rs 500/m^3
1: 2 : 4 reinforced concrete exclusive of formwork	Rs 350/m^3
highstrength deformed bars in reinforcement	Rs 3.80/kg
shuttering for foundation	Rs 12/m^2
shuttering for flights and landing	Rs 20/m^2

The cement and torsteel were issued by the department at Rs 520 per tonne and at Rs 3005 per tonne, respectively. The quantity of Ml5 concrete was 6.6m^3 which went into the foundation, while of M20 concrete which was used in the body of one staircase, i.e. connecting ground floor to first floor, was 6.60m^3. The total torsteel used was, nearly 1.80 tonnes, nearly half of which went into the top landing beam-cum-slab, which was conservatively designed along with the footing. There is certainly scope for affecting economy in the usage of reinforcement. At the quoted rates of the contractor, the structural portion of the staircase cost about Rs 13,000. Another view of the completed structure is shown in *Fig 9*.

Fig 9 Another view of the completed staircase

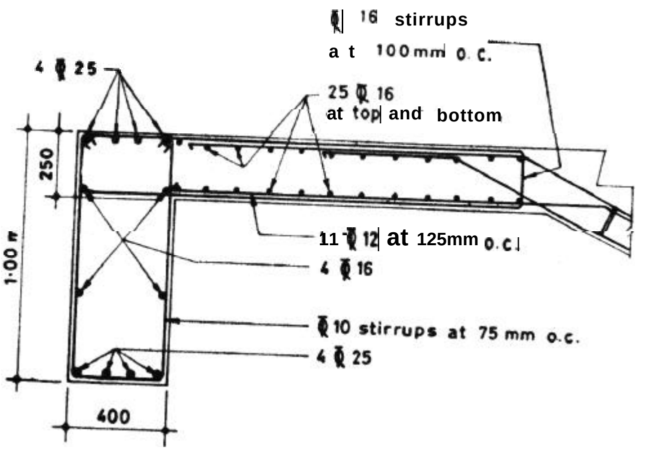


Fig 10 Cross-section of top landing beam and slab

Acknowledgement

This staircase described in the article forms part of the commercial complex of the Hindustan Shipyard Limited at Visakhapatnam, for which the author was the architect and structural engineer. The contractors were Messrs Parandhamiah and Company, of Visakhapatnam. The author is grateful to his staff, Messrs B. B. Appa Rao and G. Sriramamurthy for their help given in the preparation of the manuscript.

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APPENDIX

Detailed calculations

The analysis of a free-hanging staircase is primarily based on the premise that the stairs are symmetrically loaded with ends fixed, and with the midlanding portion treated as a proppedcantilever, giving a line support in 'symbiotic-state' to upper and lower flights.

The physical dimensions of the staircase in question are

tread	300mm
riser	150mm
width of stairs	1500mm
width of midlanding	1500mm
length of midlanding	3500mm
horizontal length of going of stairs	3300mm
waist slab thickness	200mm
landing slabs	250mm
total dead load of flight with the above dimensions	3544kg.
live load on flight	$3.3 \times 1.5 \times 500$ kgs/m²
	-2475 kgs.
Therefore, total load of the flight	=6019kgs.

Considering the support offered by mid-landing as a propped cantilever, the reaction at the free end

$= \frac{3}{8} \times 6019$kgs
$= 2257$kg

Total weight of midlanding	= 2992kg
Total live load on landing	$1.5 \times 3.5 \times 500$ kg/m² = 2625kg
Therefore, total loads of midlanding	= 5617kg
Load of landing per flight = $\frac{5617}{2}$	= 2809kg
Reaction from the flight	= 2257kg

Therefore, total reaction from upper flight causing a bending moment on the landing, about the longitudinal axis of the staircase, due to eccentricity of loads = 2809 + 2257 = 5066kg

Therefore, bending moment (hogging) on the landing = 5066xlm = 5066kg-m

Using M20 grade of concrete with $\sigma_{st} = 230$N/mm² for a balanced section, the moment of resistance is $8.98bd^2$

Therefore, $8.98bd^2 = 5066$kg-m

$$d = \left(\frac{5066}{8.98 \times 1.5}\right)^{\frac{1}{2}} = 19.39\text{cm}$$

providing 250-mm thick slab, area of reinforcement

$$= \frac{5066 \times 100}{2300 \times 0.9 \times 17.5} = 13.98\text{cm}^2$$

Thus, four 16-mm diameter torsteel plus six 12-mm diameter torsteel are provided giving an area of 14.82cm² both at top and bottom, in the landing slab of the staircase, *Fig* 10.

Since the midlanding slab, under uniform loading of the entire staircase, suffers bi-axial bending in the X-Y plane, it is necessary to provide top reinforcement in the slab, parallel to X-axis. For this purpose the midlanding shall be assumed as cantilevering out from the line of flights.

Total loads on landing per metre width $= \frac{5617}{3.5} = 1605$kgs

Moment $= 1605 \times \frac{1.5}{2} = 1204$kg-m.

Area of reinforcement $= \frac{1204 \times 100}{2300 \times 0.9 \times 22.5}$

$= 2.59$cm² per metre.

Thus 8-mm diameter torsteel at 150-mm centres, is provided giving an area of 3.35cm² per metre width of mid-landing slab, *Fig* 11.

Detailing of reinforcement in flights: The negative moments at supports due to vertical loads, causing flexural moments, till be the algebraic sum of moments $\frac{WL}{8}$, treated as a propped cantilever

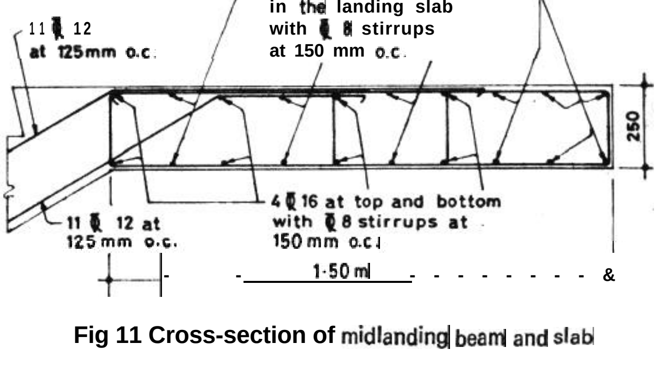

Fig 11 Cross-section of midlanding beam and slab

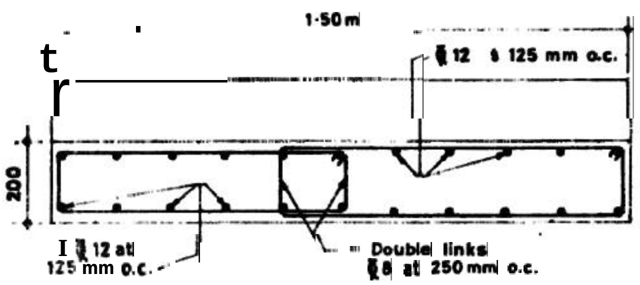

Fig 12 Transverse section of flight slab

plus the carry over moment from the cantilevered midlanding.

Therefore, negative moment $= -\frac{6019 \times 3.41}{8} + \frac{1240 \times 1.5}{2}$

$M_u = -1663$ kg-m

The above moment will be compounded with torsional moments arising from unsymmetrical live load conditions on the flights. The torsional stresses will be maximum in any one of the flights, when the other flight is fully loaded with live load.

For this condition, the reaction at the midlanding due to live load on one flight is

$$\frac{3.3 \times 1.5 \times 500 \times 3}{8} + 1.5 \times 1.75 \times 500 = 2241 \text{ kg}$$

Lever arm with respect to centre line of flight $= \frac{1.5}{2} + \frac{0.5}{2} = 1$ m

Therefore, for torsional moment, $T_u = 2241 \times 1 = 2241$ kg-m

According to clause 40.4 of IS: 456-1978, reinforcement for torsion when required, shall consist of longitudinal and transverse reinforcement. Thus

$$M_t = \frac{T_u\left(1 + \frac{D}{b}\right)}{1.7}$$

$$= 2241 \frac{\left(1 + \frac{0.20}{1.5}\right)}{1.7} = 1494 \text{ kg-m}$$

Therefore, $M_{e1} = M_u + M_t$

$= 1663 + 1494 = 3157$ kg-m

$8.98 bd^2 = 3157$ kg-m

$8.98 \times 1.5 \times d^2 = 3157$

$d = 15.3$ cm

But actual thickness of slab provided in 200mm.

Area of steel

$$= \frac{3157 \times 100}{2300 \times 0.9 \times 17}$$

$= 8.97 \text{ cm}^2$

Thus 11. 12-mm diameter torsteel provided at top in the flight slab, gives an area of 12.43 cm^2, which is O.K.

The maximum positive moment in the slab

$$= \frac{9}{128} \times 6019 \times 3.4 = 1439 \text{ kg-m.}$$

$M_t = 1494$ kg-m (as above)

Therefore positive $M_{e1} = 1439 + 1494$

$= 2933$ kg-m

Since positive moment also is more or less equal to the negative moment, same reinforcement, namely, eleven 12-mm torsteel is provided at bottom in the flight slab, Figs 12 and 13.

According to clause 40.4.3 of IS:456-1978 transverse reinforcement

$$A_{sv} = \frac{T_u S_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u S_v}{2.5\, d_1 (0.87 f_y)}$$

Shear V_u at the fixed end

$= 6019 - 2257 = 3762$ kg

$$A_{sv} = \frac{2241 \times 100 \times 25}{145 \times 15 (0.87 \times 415 \times 10.19)}$$

$$+ \frac{3762 \times 25}{2.5 \times 15 (0.87 \times 415 \times 10.19)}$$

$= 1.38 \text{ cm}^2$

Thus 8-mm torsteel four-legged stirrups provided at 250-mm centres, gives an area of 2.012 cm^2, which is OK.

Detailing of reinforcement in top landing: The top lading is subjected to the reactions from the flights emanating/culminating at it, causing flexural momenta and also torsional moments due to the eccentric loading of the flights.

The reactions from the flights spread over a length of 1.5m is 3762kg each. The clear span of the landing slab is 6m; the width of the slab is 1.7m; assumed thickness of the slab is 0.25m.

Fixed end moments, due to the reactions of 3762kg. at the ends of the top landing slab are 5307 and 3781-kg-m. respectively.

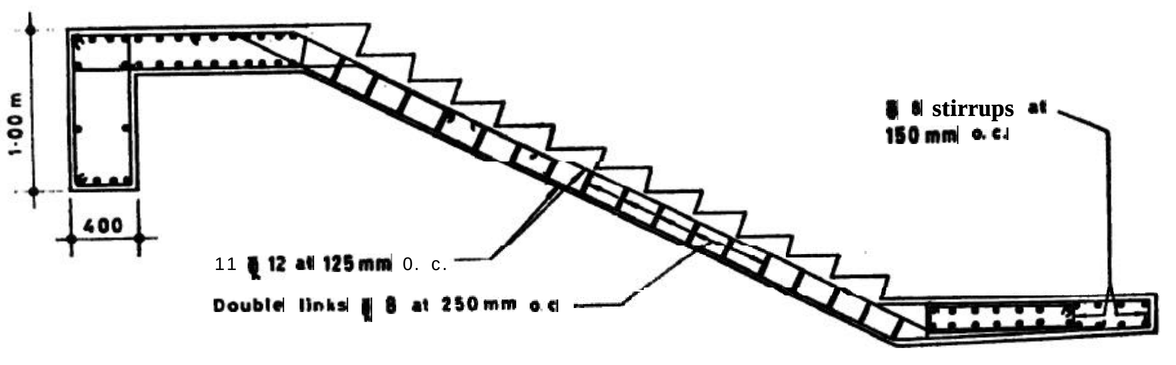

Fig 13 Cross-section of upper flight

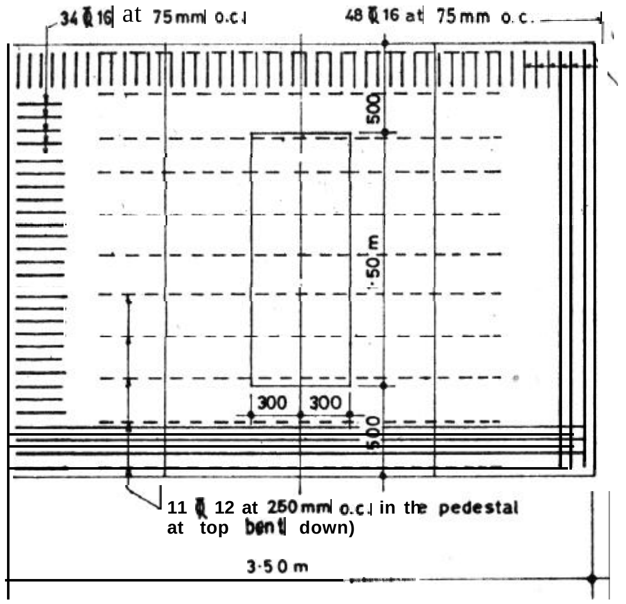

Fig 14 Details of foundation for lower flight

Loads on the landing slab

Dead load = 0.25 x 2400 $= 600 kg/m^2$

Finishes, 40mm thick $= \frac{100 kg/m^2}{700 kg/m^2}$

Live load $= \frac{500 kg/m^2}{1200 kg/m^2}$

$$FEM = \frac{WL^2}{12}$$

$$= \frac{1200 \times 6^2}{12} = 3600 kg\text{-}m.$$

Therefore, negative moment at support

= 5307 + 3600

M_u = 8907kg-m

Negative moment developed at the supports of the flights = 1663kg-m per flight. This moment acts as a torque on the landing slab.

$$M_s = \frac{T_u\left(1 + \frac{D}{b}\right)}{1.7}$$

$$= \frac{1663\left(1 + \frac{0.25}{1.7}\right)}{1.7}$$

= 1122kg-m.

Therefore, $M_{cl} = M_u + M_l$ = 8907 + 1122

= 10029kg-m.

$8.98 bd^2$ = 10029kg-m.

Therefore, $8.98 \times 1.7 \times d^2 = 10029$

Therefore, $d = 25.63 cm$.

Hence, it is required to provide 250-mm thick slab overall since doubly reinforced slab can take care of the extra moment of resistance to be developed.

$$A_s = \frac{10029 \times 100}{2300 \times 0.88 \times 22}$$

$= 22.52 cm^2$

With 220-mm effective depth, moment of resistance of the slab is = 8.98 x 1.7 x 22^2 = 7388kg-m.

For balance moment = 10029 − 7388 = 2641 kg-m

With allowable compressive stress $1900 kg/cm^2$

Lever arm = 19cm

$$A_{sc} = \frac{2641 \times 100}{1900 \times 19} = 7.32 cm^2$$

$$A_{st} = \frac{7388 \times 100}{2300 \times 0.88 \times 22} = 16.59 cm^2$$

Therefore total reinforcement = 16.59 + 7.32 = $23.91 cm^2$, say $24 cm^2$.

Foundation design: The design of foundation is to be principally based on statical considerations. The reactions from the upper flight, midlanding and from the bottom flight, which constitute the unbalanced loads, will have to be resisted by the inertial mass of the foundation block.

The detailing of reinforcement in the foundation is shown in *Figs* 14 and 15. The foundation consists of a rectangular slab, 2.5m x 3.5m x 0.55m thick with a central pedestal of 1.5m x 0.60m x 0.45m in height. The dead weight of the footing is 11.522t.

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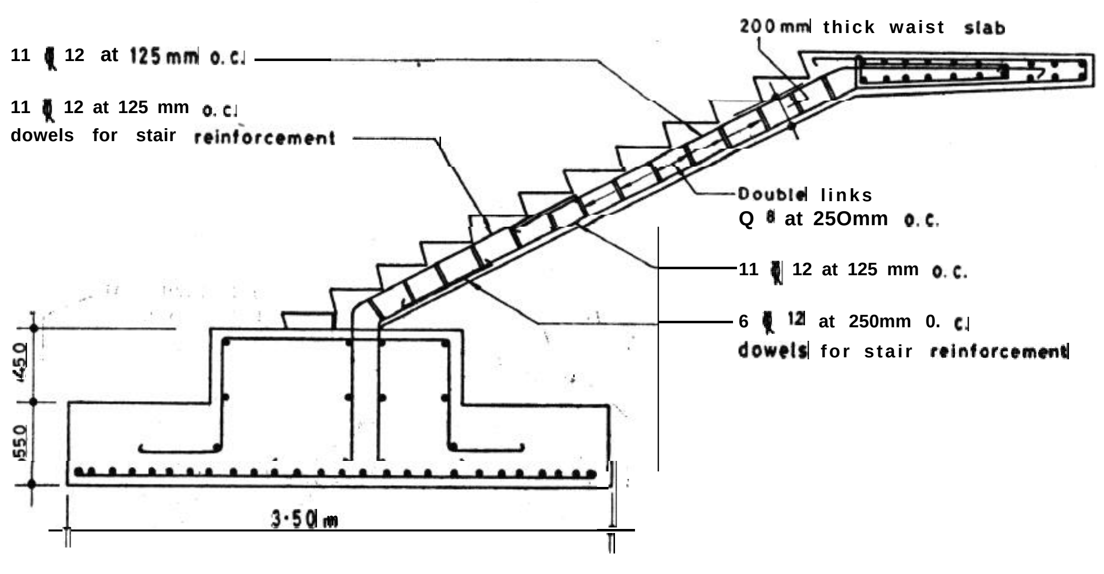

Fig 15 Cross-section of foundation and lower flight